Modeling for Equitable Food Distribution under Capacity Constraints

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Abstract

In partnership with the Food Bank of Eastern & Central North Carolina, which distributes food to a 34-county service area, the objective of this research is to determine the equitable distribution of donated food among people at risk for hunger. In this context supply is donated, demand typically exceeds supply and is proportional to the poverty population, which is also difficult to determine unambiguously at any time. Equitable food distribution allows for each individual in the considered community to have equal access to affordable, healthy and nutritious food such that no individual is at a disadvantage compared to others. We define effectiveness as the ability to distribute as much food as possible from the donated supply, which often conflicts with the objective of equity. Factors that influence the solution of this problem are capacity, supply and equity. First, we use a linear programming approach to derive structural properties. By relaxing the equity constraint, we develop an efficient frontier that illustrates the trade-off between equity and effectiveness.

Keywords
Operations research, equitable food distribution, food bank, capacitated network flow problems, efficient frontier.

1. Introduction

According to the Food and Agriculture Organization of the United Nations [1], 925 million people in the world suffered from food insufficiency and hunger in 2010. In 2010, 48.8 million Americans lived in food insecure households; 32.6 million adults and 16.2 million children [2]. The United States Department of Agriculture [3] defines food insecurity as “a household-level economic and social condition of limited or uncertain access to adequate food” and hunger as “an individual-level physiological condition that may result from food insecurity”. Although the term “hunger” was more common in the 1980s, “food security or insecurity” is the term in currently used [4]. This paper contributes to the relief of food insecurity through analysis of the operations of a large food bank aimed at equitable distribution of donated food to the population in need in North Carolina.
The Food Bank of Eastern & Central North Carolina (FBCENC) is located in Raleigh, NC and distributes food and other donations through its own warehouse and four branches (located in Wilmington, Durham, Sandhills and Greenville, NC) to partner agencies such as food pantries and soup kitchens over a 34-county service area. The food is distributed to the needy population in the county by these partner agencies. Each branch is assigned a set of counties that they send food to, and a county may receive food from more than one branch. The number of agencies in a given county can vary over time as agencies are suspended due to failure to meet legal reporting requirements. Thus the amount of food that can be distributed in a given county can be considered a random variable. An example of this is the use of mobile food pantries that operate in some periods and shut down in others. The supply chain associated with FBCENC is illustrated in Figure 1.

The supply chain associated with food donations and distribution involves complex capacity and operational constraints as well as uncertainty in both supply (donations) and demand (population in poverty in the service area). However, unlike for-profit supply chains, supply uncertainty is more critical than demand uncertainty since demand almost always exceeds supply. Supply is uncertain in both quantity and the mix of different types of food that are donated, which place quite different stresses on the supply chain. For example, even though one can generally hold inventory of dry goods for some time, perishable items like fresh produce must be distributed within a very short time of receipt or be discarded. These issues come into conflict with FBCENC’s requirement to demonstrate equitable distribution of the donations it receives to the population in need across its service area. The estimated demand for donated food plays a major role in this consideration of equity.

FBCENC’s current practice for distributing food is based on a fair-share formula, which defines the demand for donated food based on the estimated population in poverty (as defined by the Federal government) in each county. FBCENC is required to distribute food across counties in proportion to the population in poverty in each county. Thus, ideally, each person in poverty in FBCENC’s service area would receive exactly the same amount of donated food.
food by weight over a specified reporting period. However, when historic distribution data is examined, some inequitable distribution is observed; some counties are at a disadvantage compared to others in terms of obtaining their share of the donated food. In order to demonstrate this inequity, we calculated 90 percent confidence intervals for the poverty populations in all counties using data from U.S. Census.[5] We then took the actual total distribution from July 2009 to January 2010 and came up with the ranges of values that each county should have received during this period if the distribution were to be equitable. When we compared the results to actual historical data, we observed that approximately one third of the counties were underserved for at least five months of this period while another one third of the counties were overserved for at least five months. Historical records also indicate that at some locations, perishable donated food is discarded because it cannot be distributed in a timely manner while other locations are experiencing shortages at the same time. In this study we propose a deterministic network flow model that tries to maximize the amount of distributed food while maintaining equity as defined by the fair share measure within a user-specified tolerance. The model can be used both for benchmarking the performance of FBCENC by exploring the tradeoff between equity and distributed tonnage under ideal conditions, and for obtaining managerial insights into how capacity investments can be made in collaboration with local agencies to improve the ability of FBCENC to meet its goals. The deterministic models developed here will form the basis for the future development of stochastic optimization models that directly address the uncertainties in supply and demand.

In the next section, we briefly review the related literature on definition and measures for equity and equitable distribution in various applications. In Section 3, the model formulation is presented. In Section 4, the methodology for this study is discussed building on the model from the previous section and the data acquisition process will be explained. We then summarize the results of an example case and the major conclusions from this study. The paper concludes with a discussion of the limitations of the current work and directions for future research.

2. Background & Literature Review
The major issue to address related to this research is the definition of economic equity. Sen [6] states that economic inequality can be described either in an objective sense (in terms of using some statistical measures) or in terms of some normative notion, i.e. a higher degree of inequality corresponds to a lower level of social welfare for a given total of income. In this research we focus on objective inequality measures of which, some examples are mean deviation, range and standard deviation.

Fairness and equity are abstract, socio-political and subjective concepts and because of this, it is very difficult, if not impossible, to determine an equity measure that is applicable to all different types of problems. [7] There has been a wide range of applications in the literature considering the equity and equitable allocation of different types of resources. Marsh et al. [8] gave a brief overview of the many areas for which equity is an objective. Some examples of these are geographers’ concerns regarding equitable distribution of water rights in Western states, political scientists’ discussions on each state having equal representation in Congress and economists’ studies on public welfare distribution and equitable distribution of income. Marsh et al. [8] focused on facility sitting decisions and they stated that equity was obtained if each group that is affected by the facility sitting decision receives their fair share from the total effect. Their objective was to minimize inequity by using 20 different measures proposed in literature and compare and analyze them for different situations. In this study, we use one of those measures of inequity, which is minimizing the maximum deviation. However, the remaining measures can also be easily implemented in our model to observe the differences on the results.

Meng et al. [9] approached the problem of equity from a different perspective; they worked on the Continuous Network Design Problem which is basically the problem of allocating a capacity increase on the existing roads under a budget constraint. They considered equity in terms of different network users receiving equal benefit from the capacity increase in terms of their average origin – destination travel costs. They used a bilevel programming approach where equity is incorporated as a constraint.

Equitable distribution is also a widely used objective when allocation of emergency medical resources is considered. Chanta et al. [10] considered the problem of determining locations of facilities to locate ambulances such that the total envy over all demand zones, where demand zones consist of aggregated number of patients living in certain regions, is minimized. They formulated the problem as an integer programming model called the minimum $p$-Envy Location Model. The definition of envy was based on the patient’s dissatisfaction due to his distance to close ambulance facility locations; hence their approach is similar to minimizing total inequity. Due to computational difficulty, they performed a tabu search to reach solutions quickly. They stated that according to the results, the
proposed method performs well for both satisfying equitable distribution (minimizing envy) and coverage of all service area.

Vossen et al. [11] also studied an interesting equity problem. They worked with the Federal Aviation Administration and their objective was to allocate the national air space equitably such that a possible amount of delay gets distributed equitably among flights. They proposed that the ration-by-schedule gives closely related results to equitable allocation of resources and minimizes total delay. Mazumdar et al. [12] studied the multi user telecommunications network in which each user has the objective of optimizing its performance while being fair to the other users. They proposed that the Nash arbitration scheme, from Game Theory, gives a desirable and fair solution for individual users and different performance criteria. The solution obtained from this method was Pareto optimal.

Finally, Ogryczak [7] addressed the tradeoff between equity and effectiveness in resource allocation models. He explained different performance measures to achieve Pareto optimality of a solution since this solution is also efficient. Also, he stated that the max-min types of objectives give both equitable and effective solutions and can be used in some applications. There are many other studies which also consider equity as an objective. However, as stated before, the definition of equity and the performance measure that is used is highly dependent on the specific problem that the researcher is working on. Although there have been many studies with the objective of satisfying equity over some measure, to the best of our knowledge, determining the optimal allocation of food in a donations – based supply chain remains undiscovered in spite of its theoretical and social interest.

In this study, we formulate the problem as a network flow problem with capacity constraints and try to allocate donated food to minimize the deviation from a completely equitable distribution. Although, according to our knowledge, there is no study based on donated food supply chain with capacity constraints, the general ideas and methods from production supply chains provide useful insight about our problem. Toktay and Uzsoy [13] developed a machine capacity allocation problem in a semiconductor wafer fabrication facility to maximize throughput and minimize deviation from predetermined production goals. The authors showed that these two objectives are actually equivalent and both strongly NP hard. Hence, the authors developed heuristics to find near optimal solutions. Ulusoy and Uzsoy [14] worked on the strategic mobility problem which is defined as “the problem of moving a known amount of resources from a number of supply points to a number of demand points using a number of vehicles.” As a special case they considered the airlift problem and their purpose was to locate supplies and aircrafts to minimize the maximum operating time while considering possible demand patterns. They considered two levels of planning: first was the allocation of supplies and the second was taking optimal action of routing the aircraft in a particular situation. For the first level they used a mathematical model with a scenario approach and for the second level they used a heuristic approach. This two-level planning system for allocating resources can also be applied to our model in the future since the distribution occurs at both the branches and the counties. Finally, Parlar and Perry [15] considered the inventory planning problem in a production facility under supply uncertainty. They modeled this as a continuous-time Markov Chain with the states being supplier availability states. They considered a single machine problem with a single supplier, two suppliers and multiple suppliers. Although the stochastic nature of our problem is significant, we have focused on the deterministic constrained equitable allocation problem first.

3. The Model
A linear programming model is presented to obtain the optimal allocation of donated food where both equity and effectiveness are considered. To obtain insight we first develop a model based on aggregated total tonnage of a single food group such as dry goods, whose capacity constraints are relatively simple compared to, say, refrigerated foods which require consideration of the available refrigerator space in addition to agency capacities. Distribution is said to be equitable if each person in poverty in the considered service region receives the same amount of distributed supply. On the other hand, the distribution is effective if the maximum amount of supply gets distributed. Demand is considered to occur at the county level, hence the agencies in a given county are aggregated to provide estimates of the distribution capacity in that county, i.e., the total tonnage of food that the county can receive and distribute with minimal wastage. We propose the following model:

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In this problem, it is not possible to predict demand with certainty. It is reasonable to assume that demand is proportional to the poverty population in the considered service region. However, the definition of the population in poverty is somewhat arbitrary, and families and individuals enter and leave this population constantly for a variety of reasons such as relocation, job loss or finding new employment. It is important to note that satisfying demand is not possible since it significantly exceeds supply. Hence, satisfying demand is not the objective in this formulation. Demand is only used to determine the fair share for a county; the donated goods should be distributed according to

4. Methodology

4.1 Data Acquisition

FBCENC classifies the donated goods into four categories: dry goods, produce, refrigerated food and frozen food. In this study, the focus is on dry goods. In this study, the actual monthly donations made to the five branches from January 2009 to June 2009 are used as the supply data. These donations originate from many different sources such as large companies and individuals.

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the poverty populations of the counties. The poverty populations for the counties are obtained from the data from U.S. Census Bureau [5].

The capacity issue in this supply chain can be studied at different levels. In general, the flows in this network are constrained by capacities at branches and counties and transportation capacity. The branch and county capacities arise due to limited storage space. The transportation capacities occur due to limited ability to move food from one place to another and are related to number of trucks and truck capacities. In this model, the focus is on the capacities at county levels. In order to determine the capacity of a county, the actual distribution data into that county during fiscal year 2009 is taken into account. The 90th percentile of this empirical distribution data is used to represent that county’s capacity. This is a reasonable estimation since it gives an idea of what that county can absorb in general. It is assumed that this capacity is constant over the time horizon of the LP model. These assumptions have also been validated with FBCENC.

4.2 Solving the Model

The proposed model is solved using GAMS optimization software. We now discuss some structural properties..

If the tradeoff between equity and effectiveness is not incorporated into the model, trivial results are obtained. For example, if only equity is considered as an objective, a trivial solution would be to distribute nothing from the branches. In that solution, all the donations would remain unshipped and none of the counties would receive any food; which is an equitable distribution since everyone receives the same amount. On the other hand, if the objective is to distribute as much food as possible without considering equity, then, the food would just be distributed randomly among counties as much as their capacities allow. This would result in a highly inequitable distribution. Hence, it is necessary to incorporate both of these objectives into the model simultaneously. This phenomenon will be discussed further in Results section where an efficient frontier for food distribution is obtained.

5. Results

5.1 An Illustrative Case

The proposed model is solved with the data described above for the time period from January through June 2009. Only dry goods are considered. In this initial case, the deviation constant \( K \) is set equal to zero; hence every person in poverty living in the considered service region must receive the same amount of donated food. As a result, the distributed amounts are based on the receiving capacities of the agencies. The total amount of food distributed ends up being the same every month during the horizon because the capacities of the counties are the same every month. The flows are not constrained by the supply since inter-branch flows are allowed. Allowing inter-branch flows enables the transportation of food from one branch to another if it is needed. The model provides the flows from branches to counties, inter-branch flows and undistributed supply at each branch for each month in the planning horizon. Using these values the ratios of the flows into each county to its receiving capacity are calculated. The ratios for January 2009 for each county are shown in Figure 2 in decreasing order from left to right.
Figure 2 shows that the calculated ratio is equal to one for Wilson County and less than one for the remaining counties. This means that Wilson County receives the amount of food equal to its capacity, while the remaining counties receive less. The county which receives the amount of food equal to its capacity is said to be the bottleneck county. In the considered problem, Wilson County becomes the bottleneck county in each of the months for the six month planning horizon. Based on these, we can prove the following proposition.

**Proposition 1:** If deviation value \( K_t \) is set equal to zero, in an optimal solution to the model above, the county with the smallest capacity/demand ratio (the bottleneck county) will end up receiving an amount of food equal to its capacity whereas the remaining counties will receive food less than their capacity.

Next, we compute the ratio of capacity to demand for each county as illustrated in Figure 3 for all the counties in a normalized fashion such that the greatest ratio is equal to one. In Figure 3, the counties are given in an increasing order of their calculated ratios.
It is significant that the orderings of the counties in Figures 2 and 3 are the same. This is expected since equitable distribution is enforced and the ratio of the flow sent to each county to its demand is the same for all counties.

Hence we observe that the bottleneck county is actually the county with the smallest capacity to demand ratio. Since every county receives their share from the distribution in proportion to their demand, all other counties are penalized due to the low level of capacity of the bottleneck county. Hence, it is straightforward to ask the question of what happens if the capacity of the bottleneck county is increased. If the capacity of the Wilson County is increased by one pound, (but remains to be the bottleneck county), then, each county j ends up receiving $D_j / D_{Wilson}$ more food. If the capacity of Wilson County is increased to the amount such that its capacity over demand ratio is greater than that of Granville, then Granville County becomes the new bottleneck county. In other words, if we increase Wilson County’s capacity such that:

$$\frac{C_{Wilson}}{D_{Wilson}} > \frac{C_{Granville}}{D_{Granville}}$$

Then, Granville County becomes the bottleneck county which directly follows from Proposition 1.

5.2 Efficient Frontier for Dry Goods Category
The next issue to consider is the relationship between equity and effectiveness. In this section, an efficient frontier illustrating the relationship between the total undistributed supply and the deviation constant is obtained considering the total planning horizon. Again, only dry goods category is considered. First, $K_t$ is varied from zero to one in increments of 0.001. The total undistributed supply decreases as $K_t$ is increased. However, once $K_t$ exceeds a given value we obtain no additional benefit to effectiveness from increasing it further. Figure 4 shows this relationship with the section where $K_t$ is between zero and 0.03 with increments of 0.001.
From Figure 4, we see that for \( K_t \geq 0.027 \), the undistributed supply curve becomes horizontal. This shows that after this point, relaxing the equity constraint is not helpful in terms of distributing more supply. This occurs because the distribution starts to be determined according to the capacities of the counties rather than equity constraints.

### 6. Conclusions

The purpose of our study is to develop a model to determine the equitable distribution of donated food. The conflicting objectives of equity and effectiveness play an important role in this formulation where we define effectiveness as the ability to distribute as much food as possible to the counties in the service region. On the other hand, a distribution is equitable if no person at the community is at a disadvantage in terms of receiving donated food. We formulate the problem as a linear programming model and derive some structural properties. As a case study, we use the data obtained from FBCENC.

The major conclusion that we make from this study is the notion of the bottleneck county and how it affects the solutions in terms of the equity-effectiveness tradeoff. The bottleneck county is the county with the smallest capacity to demand ratio and as stated in Results section, this county ends up receiving an amount of food equal to its capacity whereas other counties get penalized because of it if we enforce a perfectly equitable distribution. Then, we develop an efficient frontier approach to visualize the conflicting objectives of equity and effectiveness. We realize that if we relax the requirement for perfectly equitable distribution, we end up distributing more supply.

There are some assumptions made in this study. First, it is assumed that the demand, supply and capacity are deterministic. This blends in with the purpose of this study since we want to understand the general behavior and structural properties of this network flow problem. This assumption will be relaxed in our future studies. Also, in the case study, we focus on one particular food group. In our future studies, we will incorporate other food groups in the model and explore how we can satisfy equity when all groups are considered together.

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